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Solution of the Navier–Stokes Equations for the Processes of Inertial Gas-Dynamic Separation in the Curvilinear Channels

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In this paper as a result of the analytical solution of the Navier-Stokes equations for gas flow in the plane semicircular annular channel the radial and circumference velocity components were determined taking into account boundary conditions, limitations and hypotheses. Equation for gas leakages determination and expression for pressure distribution were received.

Keywords: *hydrodynamics, the Navier–Stokes equations, the flow in the plane curvilinear channel, inertial gas-dynamic separation.*

1. Introduction

The processes of formation and separation of the heterogeneous dispersion system (emulsions, suspensions, aerosols) play an important role in science and technology. In terms of specific energy consumption and efficiency of separation, methods of inertial gas-dynamic and inertial-filtrating separation, which are differ in the ways of formation of the geometrical configuration of the separation channels, and the character of movement and path of flow, are considered to be optimal [1].

Traditionally, the corrugated packing blocks with sine wave or zigzag form (corner packing blocks) are set to separators. The first are widely used in the separators of domestic production, the second – in the international. In both cases, the scientific problem of hydrodynamic processes modeling aimed to predict separation efficiency, as well as development of reliable engineering design techniques of typical separation device is a topical problem.

It should be noted that the theory of isothermal fluid or gas flow is based on system of main equations of fluid dynamics: continuity and Navier-Stokes equations. Solution for the given system is one of the six Millenium problems [2]. In recent times, mathematicians and physicists are keenly discussing the main statements of this problem [3]. Also there is an opinion of the impossibility of solving of this problem with currently existing methods [4]. Analytical solutions of the equations are found only in certain special cases, for small Reynolds numbers and simple geometry of the channels (e.g., Poiseuille flow). In other cases the numerical simulation with computational fluid dynamics and finite element analysis is used.

This article studies the mathematical formulation and solution of the problem of modeling the gas-liquid flow motion in the plane curvilinear channel of the separation devices with rigid stationary walls.

2. Problem Objectives and the Mathematical Model

All of the following mathematical formulations as well as the system of nonlinear differential equations of the 2nd order have an analytical solution only in very rare cases of the simple geometry of the channels.

Hereby in this paper has been accepted such simplifications and assumptions as the plane flow along the curved channel (Navier-Stokes equations are compiled for twodimensional space in the polar coordinate system). It is expected that overflow and changing of velocity and pressure fields in channel height are insignificantly in comparison with the similar parameters through the channel length. Pressure difference over the channel width is also insignificant due to the small channel width. Significant changing of the pressure difference takes place along the channel length, herewith the curvilinear viscous flow is accompanied by the process of conversion of the mechanical energy from the potential to the kinetic and vice versa.

Isothermal gas flow in the plane semicircular annular channel (fig. 1) is considered due to the circumferential pressure gradient ($p = p(\varphi)$, $\partial p/\partial r = 0$) that is described in the polar coordinates by the continuity and Navier-Stokes equations:

$$\frac{\partial V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\varphi}{\partial \varphi} + \frac{V_r}{r} = 0;$$
$$V_r\frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{V_\varphi^2}{r} = \varepsilon\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varphi^2} - \frac{V_r}{r^2} - \frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi}\right);$$
$$V_r\frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi} - \frac{V_\varphi V_r}{r} = -\frac{1}{\rho r}\frac{dp}{d\varphi} + \qquad (1)$$
$$+\varepsilon\left(\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial V_\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_\varphi}{\partial \varphi^2} - \frac{V_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi}\right),$$

where ρ is gas density; ε is turbulent viscosity coefficient according to Boussinesq hypothesis.

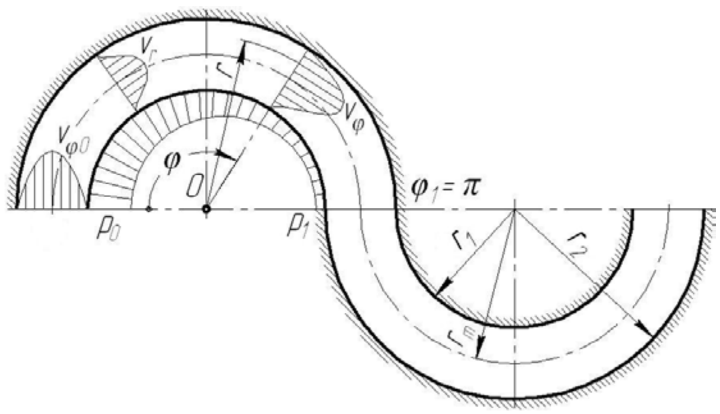

Fig. 1 – The analytical model of the channel

For given conditions at the inlet and outlet of the separation channels (an expense of a continuous phase, velocity, pressure and the stream direction) taking into account viscosity, an optimal geometric shape of the channel that provides the minimum of total pressure loss, is exists. The exact solution of a problem of optimum profiling has significant difficulties. Approximating methods based on simple physical understanding of hydrodynamically expedient distribution of the gas velocities in the flow core and near the channel walls are used in practice [1, 5]. In this case, simplification of walls profile of the curvilinear channel assuming that curvilinear sites have constants internal r_1 and external r_2 radiuses is permitted.

3. Solving Equations

The distribution functions of the radial and circumferential velocities, satisfying the continuity equation, are accepted as infinite modified power series

$$V_r = \sum_{i=2}^{\infty} \frac{dA_i(\varphi)}{d\varphi} f_i(r); V_\varphi = q\beta(r) + \sum_{i=2}^{\infty} A_i(\varphi)\psi_i(r), \ldots\ldots \quad ..(2)$$

where $A_i(\varphi)$ are functions to be further defined; $f_i(r)$ are linearly independent functions, which satisfy the boundary conditions of non-penetration of gas into the channel wall ($f_i(r_1) = f_i(r_2) = 0$); and q is the constant leakage; $\beta(r) = 6(r - r_1)(r_2 - r)/\delta^2$ is the distribution function of the circumferential velocity ($\beta(r_1) = \beta(r_2) = 0$, $\int_{r_1}^{r_2} \beta dr = 1$); δ is the radial gap. Functions $\psi_i = f_i + rdf_i/dr$ must satisfy the condition $\int_{r_1}^{r_2} \psi_i(r)dr = 1$, therefore $f_i = (r - r_1)^i(r - r_2)^i$.

Because of the small radial gap ($\delta << r$) functions A_i are determined from the equations of motion (1), which are averaged in the gap:

$$\begin{gathered} q\sum_{i=2}^{\infty} \xi_{1i} A_i^{//} - 2q\sum_{i=2}^{\infty} \xi_{2i} A_i = \kappa_1 q^2 - \varepsilon \sum_{i=2}^{\infty} \gamma_{1i}\left(A_i^{///} - 3A_i^{/}\right); \\ \frac{dp}{d\varphi} = -\frac{\rho}{\delta}\left[\sum_{i=2}^{\infty} \xi_{1i} A_i^{/} + \varepsilon\kappa_2\right] q + \varepsilon\left[\sum_{i=2}^{\infty} (\gamma_{2i} - \gamma_{1i}) A_i + 3\sum_{i=2}^{\infty} \gamma_{1i} A_i^{//}\right], \end{gathered} \quad (3)$$

where are flow coefficients $\kappa_{1,2}$, coefficients of convective inertia forces $\xi_{i1,2}$ and turbulent viscosity forces $\gamma_{i1,2}$:

$$\begin{gathered} \kappa_1 = \int_{r_1}^{r_2} \beta_i^2 dr; \ \kappa_2 = \int_{r_1}^{r_2} \left(\frac{\beta_i}{r} - r\beta_i^{//}\right) dr; \ \xi_{1i} = \int_{r_1}^{r_2} \beta_i f_i dr; \ \xi_{2i} = \int_{r_1}^{r_2} \frac{\beta_i \psi_i}{f_i} dr; \\ \gamma_{1i} = \int_{r_1}^{r_2} \frac{\beta_i}{r} dr; \ \gamma_{2i} = \int_{r_1}^{r_2} r\left(\frac{\psi_i}{f_i}\right)^{//} dr. \end{gathered} \quad (4)$$

The coefficients (4) are equal to zero for $i > 3$. Particularly for $i = 2$ the 1st equation (3) takes the form of the ordinary differential equation

$$A^{///} - \theta A^{//} - 3A^{/} + k^2\theta A = -\sigma \quad (5)$$

with constant coefficients $\theta = q\xi_1/(\varepsilon\gamma_1)$, $k^2 = 2\xi_2/\xi_1$, $\sigma = q^2\kappa_1/(\varepsilon\gamma_1)$. General solution is

$$A(\varphi) = \sum_{k=1}^{3} C_k e^{\lambda_k \varphi} - \frac{\sigma}{k^2\theta}, \quad (6)$$

where λ_k are the roots of the characteristic equation $\lambda^3 - \theta\lambda^2 - 3\lambda + k^2\theta = 0$. Integration constants C_k are determined by the conditions:

1) $A^{/}(0) = 0$ is the condition of absence of the radial velocity in the inlet section;
2) $A(0) = 0$ is the hypothesis of initial circumferential velocity profile;
3) $\lim_{\varphi \to \infty}[A(\varphi)/\varphi] = const$ is the condition of the velocity gradient limitation.

Integration of the 2nd equation (3) allows to define the pressure distribution

$$p(\varphi) = p_0 - \sum_{k=1}^{3} a_k q^k, \qquad (7)$$

where p_0 is inlet pressure; the resistance coefficients are $a_1 = \pi\rho\kappa_2/\delta$, $a_2 \approx 0$, $a_3 = \pi\rho\kappa_1\xi_1/(3\varepsilon\gamma_1\delta)$. Gas leakage is the real root of equation $\sum_{k=1}^{3} a_k q^k = \Delta p$, where Δp is the pressure difference.

Hereby, in consequence of the analytical solution of the gas motion equations, the radial and circumference velocity components were determined. Boundary conditions, limitations and hypotheses were taken into account. Equation for gas leakages determination and expression for pressure distribution were received.

4. Conclusion

Analyzing the results is shown that axisymmetric inlet gas-liquid flow along a curved channel forms a vortex in the cavities on the outer radius with period π radians, as well as the dynamic pressure pulsations. Therefore, it is advisable to give sinusoidal form to the wall, which is optimal in view of the assumption about the minimum pressure drop, excluding separated flow at the joined sections with jumpwise change of the radius of curvature. Subsequent investigations will be directed to numerical simulations of gas-dynamic separation in curved channels with flexible walls, which is associated with the solution of the hydroaeroelasticity problem for interaction of dispersed gas flow with baffle elements.

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